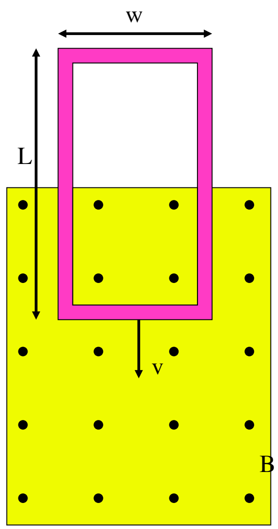
Problem 20.67

A loop of mass m, resistance R and dimension w and L falls from rest into a B-field. During the time interval before the top edge of the loop reaches the field, the loop reaches terminal velocity.

a.) What's terminal velocity?



A loop of mass m, resistance R and dimension w and L falls from rest into a B-field. During the time interval before the top edge of the loop reaches the field, the loop reaches terminal velocity.

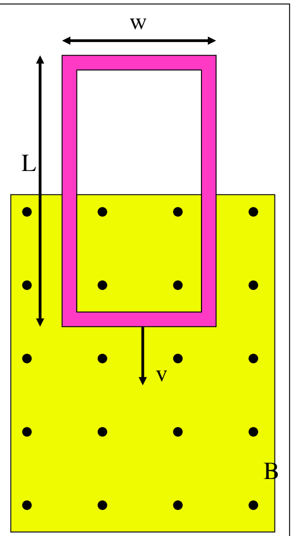
a.) What's terminal velocity?

Terminal velocity occurs when the force of gravity is exactly counteracted by the induced force generated by the induced current in the coil interacting with the external magnetic field. That is:

$$\sum F_y :$$

$$i_{induced} wB \text{ - } mg = m\cancel{a_y}^{=0}$$

To get the induced current, we have to used Faraday's Law and Ohm's Law in conjunction with one another.



In other words, with N=1 we can write:

$$i_{induced} = \frac{\varepsilon_{induced}}{R}$$

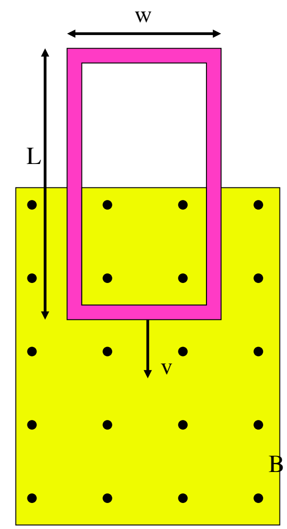
$$= \frac{-N\frac{\Delta\phi_B}{\Delta t}}{R}$$

$$= \frac{-NB\cos 0^\circ \frac{\Delta A}{\Delta t}}{R}$$

$$= \frac{-B\frac{\left[w(x+\Delta x) - wx\right]}{\Delta t}}{R}$$

$$= \frac{-Bw\frac{(\Delta x)}{\Delta t}}{R}$$

$$i_{induced} = \frac{-Bwv_{terminal}}{R}$$



Coupling the two equations

$$i_{induced} = \frac{-Bwv_{terminal}}{R} \qquad i_{induced} wB \text{ - } mg = m\cancel{a_y}^{=0}$$

we can write:

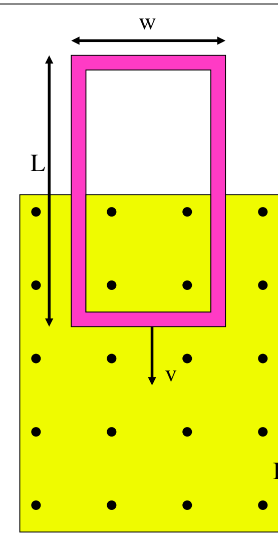
$$i_{induced} wB \text{ - } mg = m\cancel{a_y}^{=0}$$

$$\Rightarrow \left(\frac{-Bwv_{terminal}}{R}\right) wB = mg$$

$$\Rightarrow v_{terminal} = \left(\frac{mgR}{B^2w^2}\right)$$

b.) Why is the terminal velocity proportional to R?

c.) Why is the terminal velocity inversely proportional to B^2?



b.) Why is the terminal velocity proportional to R?

Bigger resistance R means smaller current i. Smaller current means less magnetic force on the wire, which means gravity holds sway and the maximum velocity is greater. That is, v is proportional to R.

c.) Why is the terminal velocity inversely proportional to B^2?

If the magnetic B gets bigger, you'd expect the current interaction with the B-field to be larger and the terminal velocity to be smaller. That means the terminal velocity and magnetic field are inversely proportional.